

# SYLLABUS

## Integrated M.Sc. in Mathematics

Applicable from the Academic Year 2010–2011



NATIONAL INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

BHUBANESWAR

Course Structure for Integrated MSc. in Mathematics

Course No.	Credits	Course Name
M101	- 3	- General Mathematics -I
M141	- 2	- Math Lab-I
M102	- 3	- General Mathematics- II
M142	- 2	- Math Lab-II
M201	- 4	- Analysis-I
M202	- 4	- Algebra-I
M203	- 4	- Discrete Mathematics
M241	- 4	- Math Lab-III
M204	- 4	- Analysis-II
M205	- 4	- Algebra-II (Linear Algebra)
M206	- 4	- Probability Theory
M207	- 4	- Elementary Number Theory
M301	- 4	- Analysis-III
M302	- 4	- Algebra-III(Rings and Modules)
M303	- 4	- Differential Equations
M304	- 4	- Topology
M305	- 4	- Statistics
M306	- 4	- Analysis-IV(Calculus of Several Variables)
M307	- 4	- Algebra-IV(Field Theory)
M308	- 4	- Complex Analysis
M309	- 4	- Optimazation Theory
M310	- 4	- Differential Geometry
M401	- 4	- Functional Analysis
M402	- 4	- Representation of Finite Groups
****	- 4	- Elective(stream)
****	- 4	- Elective(stream)
M498	- 8	- Project/Seminar
****	- 4	- Elective(stream)
****	- 4	- Elective(stream)
****	- 4	- Elective (stream)
****	- 4	- Elective (stream)
M499	- 8	- Project/Seminar
****	- 4	- Elective (stream)
****	- 4	- Elective (stream)
M598	- 16	- Dissertation
****	- 4	- Elective (stream)
****	- 4	- Elective (stream)
M599	- 16	- Dissertation

## List of Courses

### Compulsory Courses

<b>Course No.</b>	<b>Credits</b>	<b>Course Name</b>
M101	- 3	- General Mathematics -I
M141	- 2	- Math Lab-I
M102	- 3	- General Mathematics- II
M142	- 2	- Math Lab-II
M201	- 4	- Analysis-I
M202	- 4	- Algebra-I
M203	- 4	- Discrete Mathematics
M241	- 4	- Math Lab-III
M204	- 4	- Analysis-II
M205	- 4	- Algebra-II (Linear Algebra)
M206	- 4	- Probability Theory
M207	- 4	- Elementary Number Theory
M301	- 4	- Analysis-III
M302	- 4	- Algebra-III(Rings and Modules)
M303	- 4	- Differential Equations
M304	- 4	- Topology
M305	- 4	- Statistics
M306	- 4	- Analysis-IV(Calculus of Several Variables)
M307	- 4	- Algebra-IV(Field Theory)
M308	- 4	- Complex Analysis
M309	- 4	- Optimazation Theory
M310	- 4	- Differential Geometry
M401	- 4	- Functional Analysis
M402	- 4	- Representation of Finite Groups

### Elective Courses

<b>Course No.</b>	<b>Credits</b>	<b>Course Name</b>
M451	- 4	- Advanced Complex Analysis
M452	- 4	- Advanced Functional Analysis
M453	- 4	- Advanced Linear Algebra
M454	- 4	- Advanced PDE
M455	- 4	- Advanced Probability and Stochastic Process
M456	- 4	- Algebraic Geometry
M457	- 4	- Algebraic Graph Theory
M458	- 4	- Algebraic Number Theory
M459	- 4	- Algebraic Topology
M460	- 4	- Algorithm
M461	- 4	- Commutative Algebra
M462	- 4	- Cryptology
M463	- 4	- Finite Field and Its Applications
M464	- 4	- Information and Coding Theory
M465	- 4	- Mathematical Logic
M466	- 4	- Measure Theory
M467	- 4	- Nonlinear Analysis
M468	- 4	- Operator Theory
M469	- 4	- Theory of Computation
M551	- 4	- Algebraic Computation
M552	- 4	- Analytic Number Theory
M553	- 4	- Classical Groups
M554	- 4	- Ergodic Theory
M555	- 4	- Harmonic Analysis
M556	- 4	- Lie Groups and Lie Algebra
M557	- 4	- Operator Algebras
M558	- 4	- Representations of Linear Lie Groups
M559	- 4	- Representation Theory of Compact Groups

## Syllabus of Compulsory Courses

### **M101: General Mathematics-I**

Complex numbers, roots of unity, polynomials, rational functions over  $R$  or  $C$ , polar decomposition. Sequences, Limit of a sequence, convergent and divergent sequences, bounded and monotone sequences, operations on convergent sequences, limit superior, and limit inferior. Series, tests of convergence. Functions of real variable, limits, continuity, properties of continuous functions, differentiation, properties, mean value theorem, maxima and minima. Elementary functions, exponential, logarithm, sine, cosine functions, inverse functions, definition of  $x^r$ , Taylor's theorem, L'hospital's rule. Integration, Integrals of continuous functions, fundamental theorem of calculus, partial differentiation, line and surface integrals. Elementary vector calculus, gradient, divergence and curl.

*References:*

1. Tom M. Apostol, Calculus, Vol. 1: One-Variable Calculus with an Introduction to Linear Algebra, Wiley; 2 edition, 1967.
2. Thomas & Finney: Calculus.
3. Robert G. Bartle and Donald R. Sherbert , Introduction to Real Analysis, 3rd Edition, Wiley; 1999.
4. R. R. Goldberg: Methods of Real Analysis.

### **M102: General Mathematics-II**

System of linear equations, Gauss elimination method. Matrices, various types of matrices. Vector spaces, examples, sum of subspaces, basis and dimension. Linear transformations, kernel, image, the correspondence between linear transformations and matrices, rank and nullity theorem, eigenvalues, determinant. Differential equations of first and second order, Laplace transform. Discrete probabilities, combinatoric language of probabilities, conditional probabilities, Baye's formula, binomial, poisson and normal distribution and applications.

*References:*

1. K. Hoffman and R. Kunze: Linear algebra (PHI).
2. T. M. Apostol: Calculus (Vol. 2), Wiley; 2 edition, 1969.
3. G. F. Simmons: Differential Equations (McGraw-Hill), 1991.
4. E. Kreyszig: Advanced Engineering Mathematics.

### **M141: Math Lab-I**

Introduction to computers and algorithm, Languages:C/C++, Packages: Latex, MATLAB, MAPLE, MAXIMA, MATHEMATICA, GNUPlot.

*References:*

1. Brian W. Kernighan, Dennis M. Ritchie: The C Programming Language. Prentice Hall.
2. E. Balaguruswamy: Programming in ANSI C.
3. B. Stroustrup: The C++ Language. Addison-Wesley.
4. E. Balaguruswamy: Object-Oriented programming with C++.
5. T. Cormen, C. Leiserson, R. Rivest, C. Stein: Introduction to Algorithms. McGraw-Hill Science.
6. Leslie Lamport: LaTeX: A Document Preparation System. Addison-Wesley Professional 1994.
7. References for Matlab, Mathematica etc are as per instructor's recommendation.

### **M142: Math Lab-II**

Number representation and errors in computation: Representation of integers and fractions, floating point arithmetics, source of errors, loss of significance, error propagation, computation of error estimation. Solution of non-linear equations: Bisection method, secant method, Newton's method, Muller's method. Interpolations: Polynomial interpolation, Newton divided difference, cubic spline interpolation. Approximation of functions: Weierstrass and Taylor expansion, least square approximation. Numerical integration and differentiation: Trapezoidal, Simpson's and Newton-Cotes rule, Gaussian quadrature and orthogonal polynomial, numerical differentiation. Numerical solution of ODE: Euler's method, Runge-Kutta methods, multi-step formula, Predictor-Corrector methods, Boundary value methods.

*References:*

1. Kendall A. Atkinson: An introduction to numerical analysis. John Wiley and Sons.
2. S.D. Conte, C. De Boor: Elementary Numerical Analysis: An Algorithmic Approach. Tata McGraw-Hill.
3. W.H. Press et. al.: Numerical Recipes: The Art of Scientific Computing. Cambridge University press.
4. Eugene Isaacson, Herbert Bishop Keller: Analysis of Numerical Methods. Dover Publications 1994.
5. A. Iserles: A First Course in Numerical Analysis of Differential Equations. Cambridge University Press.

### **M201: Analysis-I**

Recalling convergent sequences, limit superior, limit inferior, LUB axiom of real line, open sets in  $R$ , limit points, cluster points, nested interval theorem, Cauchy sequences, completeness of  $R$ , Bolzano-Weierstrass theorem, Compact sets in  $R$ , series, properties, Cauchy Criterion, absolute convergence, Cesaro summability, real numbers and decimal expansion, differentiability, mean value theorem, Taylor's formula, derivatives of vector valued functions, Riemann integration, properties of integrals, Riemann-Stieltjes integrals, integration of vector valued functions, integration and differentiation, fundamental theorem of calculus II, change of variable formula, Functions of bounded variations, rectifiable curves.

#### *References:*

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, 3rd Edition, Wiley; 1999.
2. Walter Rudin, Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976.
3. R. R. Goldberg: Methods of Real Analysis.
4. T. M. Apostol: Calculus (Vol. 1), Wiley; 1967.

### **M202: Algebra-I (Groups and Rings)**

Set, relations, functions, Groups, subgroups, normal subgroups, cyclic groups, dihedral groups, modular arithmetic, homomorphisms, quotient groups, isomorphism theorems, General and special linear groups, permutation groups, group actions, direct products, semi-direct products, Sylow's theorems, finite  $p$ -groups, rings, ideals, fields, quotient rings, Isomorphism theorems, prime ideals, maximal ideals, nilpotent ideals, nil ideals, Chinese remainder theorem.

#### *References:*

1. I. N. Herstein: Topics in Algebra.
2. M. Artin: Algebra (PHI).
3. D. S. Dummit & R. M. Foote: Abstract Algebra (Wiley).
4. J. A. Gallian: Contemporary Abstract Algebra.
5. W. K. Nicholson: Introduction to Abstract Algebra.

### **M203: Discrete Mathematics**

Mathematical induction, Counting principles, Principles of inclusion and exclusion, Stirling numbers, Recursion, Generating functions, Partitions, Partially ordered sets, Lattices, Boolean algebra, Propositional and quantified logic, Graphs, subgraphs, Graph isomorphisms, Hamilton cycles, Euler tours, Planar graphs, Trees, Matrices associated with graphs, Matchings, Tutte's theorem, Graph coloring and Ramsey's theorem, Network flows.

*References:*

1. F. Roberts & B. Tesman: Applied Combinatorics (Pearson).
2. R. A. Brualdi: Introductory Combinatorics (Pearson).
3. G. E. Martin: Counting - The Art of Enumerative Combinatorics (Springer).
4. F. Harary: Graph Theory (Narosa).
5. G. A. Bondy & U. S. R. Murty: Graph Theory (Springer).
6. J. P. Trembely & R. Manohar: Discrete Mathematical Structure (McGraw Hill)
7. E. Mendelson: Introduction to Mathematical Logic (Chapman & Hall).
8. J. R. Shoenfield, Mathematical logic (Addison-Wesley).
9. J. Kelly: The essence of Logic (PHI).

### **M204: Analysis-II**

Metric spaces, Limit and cluster points, Dense sets, Continuity, uniform continuity, Compactness, Arzela-Ascoli Theorem, Connectedness, pathconnected sets, Complete metric spaces, completion of a metric space, Baire Category theorem, Banach contraction Principle, Sequences of functions, Pointwise convergence and uniform convergence, power series, radius of convergence, uniform convergence and Riemann integration, uniform convergence and differentiation.

*References:*

1. R. G. Bartle & D. R. Sherbert: Introduction to Real Analysis, 3rd Edition, Wiley; 1999.
2. W. Rudin: Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976.
3. R. R. Goldberg: Methods of Real Analysis.
4. T. M. Apostol: Calculus (Vol. 1), Wiley; 1967.
5. S. Kumaresan: Topology of Metric Spaces.
6. G. F. Simmons: Introduction to Topology and Modern Analysis.



### **M205: Algebra-II (Linear Algebra)**

Vector spaces, Bases and dimension, Direct sums and quotient spaces, Linear transformations and their matrix representations, Dual vector spaces, Eigenvalues and eigenvectors, Characteristic polynomial and minimal polynomial, Diagonalizations, Rational and Jordan canonical forms, Inner product and orthogonality, Quadratic forms, Bilinear forms, symmetric, hermitian forms, Unitary and normal operators, Spectral theorem.

*References:*

1. K. Hoffman and R. Kunze: Linear algebra (PHI)
2. D. S. Dummit & R. M. Foote: Abstract Algebra (Wiley).
3. A. Ramachandra Rao and P. Bhimasankaram: Linear Algebra (TRIM)
4. S. Kumaresan: Linear Algebra (PHI)
5. M. Artin: Algebra (PHI)
6. S. Lang: Introduction to Linear Algebra (Springer)

### **M206: Probability Theory**

Combinatorial probability and urn models, Independence of events, Conditional probability, Discrete and continuous sample spaces, Random variables, Distributions and density functions, Expectations, variance and moments, probability generating functions, Moment generating functions, Standard discrete distributions (uniform, binomial, Poisson, geometric, hypergeometric), Independence of random variables, Joint and conditional discrete distributions, Univariate densities and distributions, Standard univariate densities (normal, exponential, gamma, beta, chi-square, Cauchy), Expectation and moments of continuous random variables, Transformations of univariate random variables, Tchebychev's inequality and weak law of large numbers.

*References:*

1. Harold J. Larson: Introduction to Probability Theory and Statistical Inference. Wiley 1982.
2. V. K. Rohatgi: An Introduction to Probability Theory and Mathematical Statistics. John Wiley & Sons 1976.
3. K. L. Chung: Elementary Probability Theory.
4. P. G. Hoel, S. C. Port and C. J. Stone: Introduction to Probability Theory.
5. William Feller: Introduction to Probability Theory and Its Application (Vol 1 and vol. 2). Wiley.
6. G. R. Grimmett, David R. Stirzaker: Probability and Random Processes. Oxford University Press, 2001.

### **M207: Elementary Number Theory**

The unique factorisation theorem, distribution of primes, Congruences, Chinese remainder theorem, Congruences with prime-power modulus, Fermat's little theorem, Wilson's theorem, Euler function and its applications, Group of units, primitive roots, Quadratic residues, Legendre symbol, quadratic reciprocity, Arithmetic functions, Mobius Inversion formula, Dirichlet product, Sum of squares, Introduction to Zeta function.

*References:*

1. D. M. Burton: Elementary Number Theory (McGraw Hill).
2. J. A. Jones and J. M. Jones: Elementary Number Theory (Springer).
3. I. Niven, H. S. Zuckerman & H. L. Montgomery: Theory of Numbers

### **M241: Math Lab-III**

Numerical linear algebra: Solution of linear equations by eliminations, Eigen value problem, canonical forms of matrices, Solution of partial differential equations: Finite difference methods. Solving System of linear equations by conjugate gradient methods, various methods of optimization, computation of special functions, Implementation of basic Mathematical Problems: finite field, number theoretic problems (like, quadratic reciprocity, primitive roots, extended gcd), sorting and searching, graph theoretic problems.

*References:*

1. P. G. Ciarlet: Introduction to Numerical Linear Algebra and Optimisation (CUP).
2. S. D. Conte & C. De Boor: Elementary Numerical Analysis (Tata McGraw-Hill).
3. W. H. Press et. al.: Numerical Recipes - The Art of Scientific Computing (CUP).
4. K. E. Atkinson: An introduction to numerical analysis (John-Wiley).
5. J. W. Thomas: Numerical Partial Differential Equations (Springer).
6. L. N. Trefethen & David Bau: Numerical Linear Algebra (SIAM).
7. A. Iserles: A First Course in Numerical Analysis of Differential Equations (CUP).
8. T. Cormen, C. Leiserson, R. Rivest & C. Stein: Introduction to Algorithms.
9. S. Boyd & L. Vandenberghe: Convex Optimization (CUP).
10. T. Cormen, C. Leiserson, R. Rivest, C. Stein: Introduction to Algorithms (McGraw-Hill).

### **M301: Analysis-III**

Lebesgue Integral, Basic properties of Lebesgue integral, The Hilbert space of square integrable functions on  $[a, b]$ , Improper Riemann integrals, Fourier series, Convergence properties of Fourier series, Stone-Weierstrass theorem for compact spaces and locally compact spaces.

*References:*

1. G. de Barra: Measure Theory and Integration, New Age International, 1981.
2. H. L. Royden: Real Analysis, 3rd Edition, Pearson/Prentice Hall of India, 1988.
3. G. F. Simmons: Introduction to Topology and Modern Analysis. Krieger Pub Co, 2003.
4. W. Rudin: Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976.
5. R. G. Bartle & D. R. Sherbert: Introduction to Real Analysis, 3rd Edition, Wiley; 1999.

### **M302: Algebra - III((Rings and Modules)**

Field of fractions, Euclidean domains, Principal ideal domains, Unique factorization domains, Polynomial Rings, Gauss lemma, Irreducibility Criteria, Modules, submodules, quotients modules, module homomorphisms, isomorphism theorems, generators, torsion modules, direct product and sum, free modules, Tensor product of modules, finitely generated modules, exact and split exact sequences, Finitely generated modules over a PID, Structure theorem for finitely generated Abelian groups, Rational form and Jordan form of a matrix.

*References:*

1. N. Jacobson: Basic Algebra (Vol I)
2. D. S. Dummit & R. M. Foote: Abstract Algebra.
3. I. N. Herstein: Topics in Algebra.
4. S. Lang: Algebra.
5. M. Artin : Algebra.
6. Hungerford: Algebra.
7. W. K. Nicholson: Introduction to Abstract Algebra.

### **M303: Differential Equations**

Second order linear equations with constant coefficients, System of first order differential equations, equations with regular singular point, power series methods, Special ordinary differential equations from physics, Special functions like Bessel's function, Legendre polynomial, gamma function, Picard's theorem on existence and uniqueness of solution to first order ordinary differential equation, Oscillations - Sturm Liouville theory, First order partial differential equations, The Laplace, Heat and the Wave equations.

*References:*

1. S. L. Ross, Differential Equations, 3rd Edition, Wiley, 1984.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall, 1995.
3. Earl A. Coddington and Norman Levinson ,Theory of Ordinary Differential Equations, Krieger Pub Co, 1984
4. Lawrence C. Evans, Partial Differential Equations, AMS, 1998.
5. Fritz John: Partial Differential Equations. Springer 1995.

### **M304: Topology**

Topological spaces, quotient spaces, Separation axioms, Urysohn's lemma, Tietze's extension theorem, Filters and nets, Connectedness and compactness, Covering spaces, Fundamental groups

*References:*

1. T James Munkres , Topology, Second Edition, Prentice Hall, 2000.
2. K Jänich. Topology, Springer, 1984.
3. M A Armstrong. Basic Topology. Springer, 1983.
4. G E Bredon, Topology and Geometry, Springer GTM 139, 1995.
5. W. S. Massey, A basic course in algebraic topology, Springer-GTM 127, 1991.
6. I.G. Simmons: Introduction to Topology and Modern Analysis.

### **M305: Statistics**

Descriptive Statistics; Sampling Distributions, Graphical representation of data, Basic distributions, properties, Distribution theory for transformations of random vectors, Sampling distributions based on normal populations  $t$ , chi-square and  $F$  distributions. Theory and Methods of Estimation and Hypothesis testing, Point and interval estimation, Bayesian methods, Moment methods, Least squares, Maximum likelihood estimation, Criteria for estimators, UMVUE, Large sample theory: Consistency, asymptotic normality, Confidence intervals, Elements of hypothesis testing, Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests.

*References:*

1. Harold J. Larson: Introduction to Probability Theory and Statistical Inference. Wiley 1982.
2. V. K. Rohatgi: Introduction to Probability Theory and Mathematical Statistics(John-Wiley).
3. M. Miller, J. E. Freund i& I. Miller: John E. Freunds Mathematical Statistics with applications (Prentice-Hall)

### **M306: Analysis-IV(Calculus of Several Variables)**

Differentiability of functions from an open subset of  $R^n$  to  $R^m$ , properties, chain rule, partial and directional derivatives, Continuously differentiable functions, Inverse function theorem, Implicit function theorem, Interchange of order of differentiation, Taylor's series, Extrema of a function, Extremum problems with constraints, Lagrange multiplier method with applications, Integration of functions of several variables, Partition of Unity, Change of variable formula (without proof) with examples of applications of the formula, spherical coordinates, A brief introduction of Differential forms, Simplexes and Chains, Stokes theorem without proof, Deriving Green's theorem, Gauss theorem and classical stokes theorem.

*References:*

1. Wendell Fleming: Functions of Several Variables. Springer 1987.
2. Tom M. Apostol, Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra with Applications, Wiley; 2 edition , June 1969.
3. Michael Spivak, Calculus On Manifolds: A Modern Approach To Classical Theorems Of Advanced Calculus, Westview Press, 1971
4. Walter Rudin ,Principles of Mathematical Analysis, Third Edition, McGraw-Hill; 1976 .

### **M307: Algebra-IV (Field Theory)**

Solvable groups, nilpotent groups, Field extensions, Ruler and compass constructions, algebraic extensions, splitting fields, Finite fields, algebraic closures, Normal extensions, Separable and inseparable extensions, Automorphism groups and fixed fields, Galois extions, Fundamental theorem of Galois theory, Fundamental theorem of algebra, Solvability by radicals, Cyclotomic polynomials and extensions.

*References:*

1. D. S. Dummit & R. M. Foote: Abstract Algebra.
2. S. Lang: Algebra.
3. J. A. Gallian: Contemporary Abstract Algebra.
4. M. Artin: Algebra.
5. W. K. Nicholson: Introduction to Abstract Algebra.
6. Hungerford: Algebra.

### **M308: Complex Analysis**

The Cauchy-Riemann equations, Power series and holomorphy, Line integrals, the exponential map and the logarithm, Cauchy integral formula and its consequences, Cauchy's theorem, Zeros, poles and singularities of holomorphic functions, The open mapping theorem, The argument principle, Maximum modulus principle, Schwarz lemma, Residues and the residue calculus.

*References:*

1. T. W. Gamelin: Complex Analysis (Springer).
2. E. M. Stein & Rami Shakarchi: Complex Analysis, Princeton University Press, 2003.
3. R. V. Churchill & J. W. Brown: Complex Variables and Applications (McGraw).
4. L. V. Ahlfors: Complex analysis (McGraw-Hill).
5. W. Rudin: Real and complex analysis. McGraw-Hill Book Co., 1987.
6. J. B. Conway: Functions of one complex variable (Springer).
7. R. Remmert: Theory of Complex Functions. Springer 1998.

### **M309: Optimization Theory**

Linear programming, Simplex method, Duality, Transportation problems, Game theory, Calculus in normed linear spaces, Quadratic programming, Conjugate gradients, Farkas - Minkowski lemma, Kuhn-Tucker condition, Calculus of variations: Euler-Lagrange equation and some sufficient conditions, Isoperimetric inequality

*References:*

1. Frank H. Clarke: Optimization and Nonsmooth Analysis. Society for Industrial Mathematics 1987.
2. David G. Luenberger: Introduction to linear and non-linear programming.
3. Mokhtar S. Bazaraa, Hanif D. Sherali, C. M. Shetty : Nonlinear Programming: Theory and Algorithms. Wiley-Interscience 2006.
4. Bruce van Brunt: The Calculus of variations. Springer 2003.
5. S. Tijs: Introduction to Game Theory.

### **M310: Differential Geometry**

Curves in two and three dimensions, Curvature and torsion for space curves, Existence theorem for space curves, Serret-Frenet formula for space curves, Inverse and implicit function theorems, Jacobian theorem, Surfaces in  $R^3$  as 2-dimensional manifolds, Tangent spaces and derivatives of maps between manifolds, First fundamental form, Orientation of a surface, Second fundamental form and the Gauss map, Mean curvature and scalar curvature, Differential forms, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

*References:*

1. M. P. Do Carmo: Differential Geometry of Curves and Surfaces. Prentice Hall 1976.
2. M. P. Do Carmo: Differential Forms and Applications. Springer 2000.
3. John A. Thorpe: Elementary Topics in Differential Geometry. Springer 1994.

### **M401: Functional Analysis**

Normed linear spaces and continuous linear transformations, Hahn-Banach theorem (analytic and geometric versions), Baire's theorem and its consequence - three basic principles of functional analysis (open mapping theorem, closed graph theorem and uniform boundedness principle), Computing the dual of wellknown Banach spaces, Hilbert spaces, Riesz representation theorem, Adjoint operator, Compact operators, Spectral theorem for self adjoint compact operators

*References:*

1. W. Rudin, Functional analysis. McGraw-Hill, Inc., 1991.
2. J. B. Conway, A course in functional analysis. Graduate Texts in Mathematics, 96. Springer-Verlag, 1990.
3. K. Yosida, Functional analysis. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.

### **M402: Representation of Finite Groups**

Semisimple algebras and modules, Wedderburns structure theorem, Group algebras, Maschkes Theorem, Schurs lemma, Simple Modules over Group Algebras, Characters, Inner product of Characters, Character table and Orthogonality relations, Lifting a character, Induced modules and characters, Frobenius reciprocity, Algebraic integers, Real representations, Burnside's two-prime theorem, Brauers theorem on induced characters, Character table of some well-known groups.

*References:*

1. James and Liebeck: Representations and Characters of Groups.
2. J. P. Serre: Linear Representations of Finite Groups. Springer 1996.
3. J. L. Alperin & R. B. Bell: Groups and Representations (Springer).
4. B. Simon: Representations of Finite and Compact Groups.
5. N. Jacobson: Basic Algebra (Vol II)
6. William Fulton, Joe Harris: Representation Theory: A First Course. Graduate Texts in Mathematics, springer.

## Syllabus of Elective Courses

### **M451: Advanced Complex Analysis**

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates, power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Normal families, Arzela's theorem. Product developments, functions with prescribed zeroes and poles, Hadamard's theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations. Introduction to functions of several complex variables (if time permits)

*References:*

1. L. V. Ahlfors, Complex analysis. An introduction to the theory of analytic functions of one complex variable. McGraw-Hill Book Co., 1978.
2. J. B. Conway, Functions of one complex variable. II. Graduate Texts in Mathematics, 159. Springer-Verlag, 1995.
3. W. Rudin, Real and complex analysis. McGraw-Hill Book Co., 1987.
4. Reinhold Remmert: Theory of Complex Functions. Springer 1998.

### **M452: Advanced Functional Analysis**

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounov's theorem). Extreme points and Krein-Milman theorem. In addition, one of the following topics: (a) Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein-Smulian Theorem. Schauder Basis. (b) Banach algebras, spectral radius, maximal ideal space, Gelfand transform. (c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

*References:*

1. N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space. Interscience Publishers John Wiley & Sons 1963.
2. Walter Rudin, Functional analysis. Second edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., 1991.
3. K. Yosida, Functional analysis. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.
4. J. Diestel and J. J. Uhl, Jr., Vector measures. Mathematical Surveys, No. 15. American Mathematical Society, 1977.

### **M453: Advanced Linear Algebra**

Review of basic Linear Algebra, Canonical factorization, Q- forms, Bilinear forms, Hermitian forms, Duality, Tensor product, Courant- Fishcher minimax and related theorems, Perron-Frobenius theory, Matrix Norm, Matrix stability and Inequality, Generalized inverse.

*References:*

1. Steven Roman: Advanced Linear Algebra (GTM).
2. R. A. Horn & C. R. Johnson: Matrix Analysis.
3. R. Bhatia: Matrix Analysis.
4. K. Hoffman and R. Kunze: Linear Algebra.

### **M454: Advanced PDE**

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewys example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems interior regularity, local existence.

*References:*

1. G. B. Folland, Introduction to partial differential equations. Princeton University Press, 1995.
2. J. Rauch, Partial differential equations. Graduate Texts in Mathematics, 128. Springer-Verlag, 1991.
3. E. DiBenedetto, Partial differential equations. Birkhuser Boston, Inc., 1995.
4. L. C. Evans, Partial differential equations. Graduate Studies in Mathematics, 19. American Mathematical Society, 1998.
5. Michael Renardy, Robert C. Rogers: Introduction to Partial Differential Equations. Springer 2004.



### **M455: Advanced Probability and Stochastic Process**

Discrete-time Discrete-state Markov Chains, Classification of States, Recurrence, Transience, Stationary Distribution and Stability, Ergodicity, Reversibility. Topics From: Rates of convergence to stationarity, Dirichlet Form and Spectral gap methods, Some Coupling methods with applications, Random walk on Finite Groups, Poisson Processes, Continuous time Markov Chains, Birth-and-death processes, Stationary processes, Markov processes and generators, Weak Convergence of probability measures on polish spaces including  $C[0, 1]$ , Brownian motion; construction, simple properties of paths, Poisson processes, Connections between Brownian Motion / Diffusion and PDEs.

#### *References:*

1. S. M. Ross, Stochastic processes. John Wiley & Sons, Inc., 1996.
2. R. N. Bhattacharya and E. C. Waymire, Stochastic processes with applications. A Wiley-Interscience Publication. John Wiley & Sons, Inc., 1990.
3. E. Gin, G. R. Grimmett and L. Saloff-Coste, Lectures on probability theory and statistics. Lecture Notes in Mathematics, 1665. Springer-Verlag, 1997.
4. P. Billingsley, Convergence of probability measures. John Wiley & Sons, Inc., 1999.
5. K. Ito, Stochastic processes. Lecture Notes Series, No. 16 Matematisk Institut, Aarhus Universitet, Aarhus 1969.
6. D. Revuz and M. Yor, Continuous martingales and Brownian motion. Grundlehren der Mathematischen Wissenschaften, 293. Springer-Verlag, 1999
7. Geoffrey R. Grimmett, David R. Stirzaker: Probability and Random Processes. Oxford University Press, 2001.

### **M456: Algebraic Geometry**

Polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert Nullstellensatz, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezouts theorem, Abelian differential, RiemannRoch for curves.

#### *References:*

1. C. Musili, Algebraic geometry for beginners. Texts and Readings in Mathematics, 20. Hindustan Book Agency, 2001.
2. W. Fulton, Algebraic curves. An introduction to algebraic geometry. Advanced Book Classics. Addison-Wesley Publishing Company, Advanced Book Program, 1989.
3. R. Shafarevich, Basic algebraic geometry. 1. Varieties in projective space. Springer-Verlag, 1994.
4. J. Harris, Algebraic geometry. A first course. Graduate Texts in Mathematics, 133. Springer-Verlag, 1995.
5. M. Reid, Undergraduate algebraic geometry. London Mathematical Society Student Texts, 12. Cambridge University Press, 1988.

### **M457: Algebraic Graph Theory**

Introduction, Spectrum of a graph, Complexity and determinant expansions, Colouring and the spectrum, The Laplacian of a graph, General properties of graph automorphisms, Transitive and arc-transitive graphs, The spectrum and the group of automorphism.

*References:*

1. N. Biggs: Algebraic Graph Theory . Cambridge Mathematical Library.
2. C. Godsil, G. Royle: Algebraic Graph Theory. Springer International Edition.

### **M458: Algebraic Number Theory**

Norms, traces and discriminants, Algebraic number fields, Dedekind domain, Unique factorization, Quadratic fields , biquadratic fields, The ideal class group, Cyclotomic extensions, The Dirichlet unit theorem, Local field.

*References:*

1. S. Alaca, K. S. Williams: Introductory Algebraic Number Theory. Cambridge.
2. Serge Lang: Algebraic Number Theory. GTM, Springer.
3. A. Frohlich, M. J. Taylor: Algebraic Number Theory. Cambridge studies in advance Mathematics 27.
4. H. Hesse: Number Theory. Springer-Verlag, 1980.

### **M459: Algebraic Topology**

Singular homology functors and its axiomatic properties, Relations between fundamental group and first homology, Mayer-Vietoris sequence, computation of homology of spheres. Degree of maps with applications to spheres, Simplicial CW-complexes, cellular description of homology, comparison with singular theory. Computation of homology of projective spaces, Definition of singular cohomology, its fundamental properties, statement of universal coefficient theorem, Betti number and Euler characteristic, cup product, Poincare duality.

*References:*

1. M. J. Greenberg, Lectures on algebraic topology. W. A. Benjamin, Inc., 1967.
2. J. R. Munkres, Elements of algebraic topology. Addison-Wesley Publishing Company, 1984.
3. R. Bott and L. W. Tu, Differential forms in algebraic topology. Graduate Texts in Mathematics, 82. Springer-Verlag, 1982.
4. Allen Hatcher, Algebraic Topology, Cambridge University Press; 2001.

### **M460: Algorithm**

Preliminaries: Introduction to algorithms, growth of functions, time and space complexity measures, overview on data structures and algorithm design principles, Sorting and searching: Searching maximum, minimum, kth largest element in a [ordered-]list, binary search, bubble sort, divide and conquer, heap sort, quick sort, merge sort, radix sort, their time complexity, Dynamic Programming: Matrix-chain multiplication, elements of dynamic programming, longest common subsequence, Graph algorithms: Graph searching-BFS, DFS, shortest first search, topological sort; connected and bi-connected components; spanning tree- the algorithms of Kruskal and Prim, Algebraic algorithms: Winograd's and Strassen's matrix multiplication algorithm, evaluation of polynomials, DFT, FFT, efficient FFT implementation, String processing: String searching and pattern matching, Knuth-Morris -Pratt algorithm and analysis, Computational geometry: Line-segment properties, intersection of any pair of segments, finding the convex hull, finding the closest pair of points, NP-completeness: deterministic and non-deterministic algorithm, P and NP class, some NP-complete problems.

#### *References:*

1. Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman: Design and Analysis of Computer Algorithms. Addison-Wesley, 1974.
2. Thomas Cormen, Charles Leiserson, Ronald Rivest: Introduction to Algorithms. PHI, 1998.
3. E. Horowitz, S. Sahni: Fundamental of Computer Algorithms. Galgotia publication, 1987.
4. D.E. Knuth: The art of Computer Programming, Vol 1, vol. 2, vol 3. Addison-Wesley.

### **M461: Commutative Algebra**

Commutative rings, ideals, prime and maximal ideals, Noetherian Artinian ring, Prime decomposition and associate primes, Integral extensions, Valuation rings, Completion, Dimension theory, Exact sequences, Completions, Dimension theory, Cohen-Macaulay rings.

#### *References:*

1. M. F. Atiyah and I. G. Macdonald: Introduction to Commutative Algebra.
2. R. Y. Sharp: Steps in Commutative Algebra. LMS.

### **M462: Cryptology**

Overview of Cryptology: Cryptography and cryptanalysis, some simple cryptosystems (e.g., shift, substitution, affine, knapsack) and their cryptanalysis, classification of cryptosystems, concept of stream and block ciphers, , classification of attacks, Information Theoretic Ideas: Perfect secrecy, Shannon's theory, entropy, unicity distance, Stream cipher: LFSR based stream cipher, nonlinear and filter combiner model of stream cipher, RC4, Attacks on stream cipher (e.g., correlation attack, algebraic attacks etc), Block cipher: DES, linear and differential cryptanalysis, AES, Public-key cryptosystem: RSA with implementation, primality testing, integer factorization, El Gamal public-key cryptosystem, Discrete logarithm problem, elliptic curve cryptography, Digital signature and hash functions: Hash functions, security of hash functions, construction of hash functions, message authentication code, security for signature scheme, El Gamal signature scheme, Secret sharing: Shamir's threshold scheme, general access structure and secret sharing.

#### *References:*

1. A. J. Menezes, S. A. Vanstone, and P. C. Van Oorschot. Handbook of Applied Cryptography. CRC Pr Llc, 1996.
2. D. R. Stinson. Cryptography: Theory And Practice. CRC Pr Llc, 2006.
3. W. Stallings. Cryptography and network security. Prentice Hall, 2005.

### **M463: Finite Field and Its Applications**

Structure of finite fields: Characterization, traces, norms and bases, Roots of irreducible polynomial, unity and cyclotomic polynomial, representation of elements of finite field, Polynomials over finite field: Order of polynomials and primitive polynomial, construction of irreducible polynomial, binomials and trinomials, factorization of polynomials over small and large fields, calculation of roots of polynomials, Linear recurring sequences: LFSR, characteristic polynomial, minimal polynomial, Berlekamp-Messey algorithm, Applications of finite field: Applications in cryptography, coding theory, finite geometry, combinatorics.

#### *References:*

1. I.R. Lidl, H. Neiderreiter: Finite fields. Cambridge university press, 2000.

#### **M464: Information and Coding Theory**

Information Theory: Entropy, Huffman coding, Shannon-Fano coding, entropy of Markov process, channel and mutual information, channel capacity, Error correcting codes: Maximum likelihood decoding, nearest neighbour decoding, linear codes, generator matrix and parity-check matrix, Hamming bound, Gilbert-Varshamov bound, binary Hamming codes, Plotkin bound, nonlinear codes, Reed-Muller codes, Cyclic codes, BCH codes, Reed-Solomon codes

*References:*

1. San Ling, Chaoping Xing: Coding Theory: A First Course. Cambridge Univ Pr.
2. Richard W. Hamming: Coding and Information Theory. Prentice Hall, 1986.
3. Vera Pless: Introduction to the Theory of Error-Correcting Codes. Wiley-Interscience.
4. Neil J. A. Sloane, Florence Jessie MacWilliams: Theory of Error Correcting Codes, Vol I and II. North-Holland, 1983.
5. S. Lin: An Introduction to Error-Correcting Codes. Prentice Hall, 1970.

#### **M465: Mathematical Logic**

Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem. First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g. , theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godel's first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

*References:*

1. J. R. Shoenfield, Mathematical logic. Addison-Wesley Publishing Co., 1967.
2. E. Mendelson: Introduction to Mathematical Logic. Chapman & Hall, 1997.

#### **M466: Measure Theory**

$\sigma$ -algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions,  $L_p$ -spaces, Riesz representation theorem for the space  $C[0, 1]$ .

*References:*

1. G. De Barra, Measure theory and integration.
2. J. Neveu, Mathematical foundations of the calculus of probability. Holden-Day, Inc., 1965.
3. I. K. Rana, An introduction to measure and integration. Narosa Publishing House.
4. P. Billingsley, Probability and measure. John Wiley & Sons, Inc., 1995.
5. W. Rudin, Real and complex analysis. McGraw-Hill Book Co., 1987.
6. K. R. Parthasarathy, Introduction to probability and measure. The Macmillan Co. of India, Ltd., 1977.

### **M467: Nonlinear Analysis**

Calculus in Banach spaces, inverse and multiplicit function theorem, fixed point theorems of Brouwer, Schauder and Tychonoff, fixed point theorems for nonexpansive and set-valued maps, predegree results, compact vector fields, homotopy, homotopy extension, invariance theorems and applications.

*References:*

1. S. Kesavan, Nonlinear Functional analysis. Hindustan Book Agency, 2004.

### **M468: Operator Theory**

Compact operators on Hilbert Spaces. (a) Fredholm Theory (b) Index,  $C^*$ -algebras - noncommutative states and representations, Gelfand-Neumark representation theorem, Von-Neumann Algebras; Projections, Double Com-mutant theorem,  $L^\infty$  functionalCalculus, Toeplitz operators

*References:*

1. W. Arveson, An invitation to  $C^*$ -algebras. Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space. Interscience Publishers John Wiley i& Sons 1963.
3. R. V. Kadison and J. R. Ringrose, Fundamentals of the theory of operator algebras. Vol. I.Elementary theory. Pure and Applied Mathematics, 100. Academic Press, Inc., 1983.
4. V. S. Sunder, An invitation to von Neumann algebras. Universitext. Springer-Verlag, 1987.

### **M469: Theory of Computation**

Automata and Languages: Finite automata, regular expression, [non-]regular languages, deterministic and non-deterministic automata, minimiza-tion of finite automata, pumping lemma and its applications. Context free grammaers, [non-]context free languages, Chomsky normal form, push down automata, pumping lemma for CFL, Computibility: Computable functions, primitive and recursive functions, universality, halting problem, recursive and recursively enumerable sets, diagonalization, reducibility, Rice's theorem and its application, Turing machine and its variants, Churh-Turing thesis, Complexity: Time complexity of Turing machines, Classes P and NP, NP completeness, other time classes, the time hierarchy.

*References:*

1. John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Automata Theory, Languages, and Computation. Addison Wesley 2006.
2. Harry Lewis, Christos H. Papadimitriou: Elements of the Theory of Computation. Prentice Hall 1997.
3. Michael Sipser: Introduction to the theory of computation. PWS Publishing 1997.

### **M551: Algebraic Computation**

Algorithms on polynomials: GCD, Barlekamp-Massey algorithm, factorization of polynomials over finite field, lattice reduction, factorization of polynomials over  $\mathbb{Z}$  and  $\mathbb{Q}$ , Matrix Computation: Asymptotically fast matrix multiplication algorithms, symbolic and exact solutions of linear systems, Diaphontine analysis, normal forms over fields., Solving Systems of non-linear equations: Groebner basis, Buchberger's algorithms, Complexity of Groebner basis computation, F4, F5, Algorithms for algebraic number theory: Representation and operations on algebraic numbers, trace, norm, charecteristic polynomial, discriminant, integral bases, polynomial reduction, computing maximal order, algorithms for quadratic fields, Elliptic curves: Implementation of elliptic curve, algorithms for elliptic curves.

*References:*

1. Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman: Design and Analysis of Computer Algorithms. Addison-Wesley, 1974.
2. H. Cohen: A course in Computational Number Theory. Springer-verlag, 1993.
3. D.A. Cox, J.B. Little, D. O'shea: Ideals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra, Springer-verlag, 1996.

### **M552: Analytic Number Theory**

Averages of mathematical functions, distribution of primes, weyl's, Kronecker's and Minkowski's theorems, characters, Reimann Zeta function and Dirichlet L-functions, Dirichlets Theorem on primes in arithmetic progression, Functional equation and Euler product for L-functions, analytic proof of the prime number theorem. Hesse-Minkowski theorem.

*References:*

1. J. P. Serre: A course in Arithmetic. Springer-verlag, 1973.
2. H. Hesse: Number Theory. Springer-Verlag, 1980.

### **M553: Classical Groups**

The general and special linear groups, bilinear forms, Symplectic groups, symmetric forms, quadratic forms, Orthogonal geometry, orthogonal groups, Clifford algebras, Hermitian forms, Unitary spaces, Unitary groups.

*References:*

1. L. C. Grove: Classical Groups and Geometric Algebra (AMS)
2. E. Artin: Geometric Algebra (Wiley-Interscience)

### **M554: Ergodic Theory**

Measure preserving systems; examples: Hamiltonian dynamics and Liouville's theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product, Poincaré Recurrence lemma: Induced transformation: Kakutani towers: Rokhlin's lemma. Recurrence in Topological Dynamics, Birkhoff's Recurrence theorem, Ergodicity, Weak-mixing and strong-mixing and their characterizations, Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyl's theorem, The Isomorphism problem; conjugacy, spectral equivalence, Transformations with discrete spectrum, Halmos-von Neumann theorem, Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-Millan-Breiman Theorem, Flows. Birkhoff's ergodic Theorem and Wiener's ergodic theorem for flows. Flows built under a function.

#### *References:*

1. Peter Walters, An introduction to ergodic theory. Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
2. Patrick Billingsley, Ergodic theory and information. Robert E. Krieger Publishing Co., 1978.
3. M. G. Nadkarni, Basic ergodic theory. Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
4. H. Furstenberg, Recurrence in ergodic theory and combinatorial number theory. Princeton University Press, 1981.
5. K. Petersen, Ergodic theory. Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

### **M555: Harmonic Analysis**

Fourier transforms, the Schwartz space, Distribution and tempered distribution, Fourier Inversion and Plancherel theorem. Fourier analysis on  $L^p$  spaces. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem for distribution. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem.

#### *References:*

1. Y. Katznelson, An introduction to harmonic analysis. Dover Publications, Inc., New York, 1976.
2. E. Hernández and G. Weiss, A first course on wavelets. Studies in Advanced Mathematics. CRC Press, 1996.
3. Elias M. Stein and Guido Weiss: Introduction to Fourier Analysis on Euclidean Spaces (PMS-32).



## M556: Lie Groups and Lie Algebra

Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras e.g. Complex groups:  $GL(n, C)$ ,  $SL(n, C)$ ,  $SO(n, C)$ , Groups of real matrices in those complex groups:  $GL(n, R)$ ,  $SL(n, R)$ ,  $SO(n, R)$ , Isometry groups of Hermitian forms  $SO(m, n)$ ,  $U(m, n)$ ,  $SU(m, n)$ . Finite dimensional representations of  $su(2)$  and  $SU(2)$  and their connection. Exhaustion using the lie algebra  $su(2)$ , Lie algebras in general, Nilpotent, solvable, semisimple Lie algebra, ideals, Killing form, Lies and Engels theorem. Universal enveloping algebra and Poincare-Birkhoff-Witt Theorem (without proof), Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/ reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition  $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ , examples of  $\mathfrak{k}$  and  $\mathfrak{p}$  for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartans criterion for semisimplicity. Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition (statement only). If time permits, one of the following topics: (a) A brief introduction to Harmonic Analysis on  $SL(2, R)$ . (b) Representations of Compact Lie Groups and Weyl Character Formula. (c) Representations of Nilpotent Lie Groups.

### References:

1. J. E. Humphreys, Introduction to Lie algebras and representation theory. Graduate Texts in Mathematics, 9. Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari, A first course on representation theory and linear Lie groups. University Press, 2000.
3. Serge Lang,  $SL(2, R)$ . Graduate Texts in Mathematics, 105. Springer-Verlag, 1985.
4. W. Knapp, Representation theory of semisimple groups. An overview based on examples. Princeton Mathematical Series, 36. Princeton University Press, 1986.
5. Brian C. Hall: Lie Groups, Lie Algebras, and Representations: An Elementary Introduction. Springer 2004.

### **M557: Operator Algebras**

Involutive Banach algebras,  $C^*$ -algebras, Examples, Spectrum and Functional Calculus in  $C^*$ -algebras, Continuity of Homomorphisms, Positive cones of  $C^*$ -algebras, Approximate identities in  $C^*$ -algebras, Quotient algebras of  $C^*$ -algebras, Representations and Positive linear functionals, Extreme points of the Unit ball of a  $C^*$ -algebra, Banach spaces of operators on a Hilbert space, Locally convex topologies in the algebra of bounded operators of a Hilbert space, The double commutation theorem of John Von Neumann, Kaplansky's Density theorem.

*References:*

1. J. Dixmier:  $C^*$ -algebras (North-Holland), 1977.
2. S. Sakai:  $c^*$ -algebras and  $w^*$ -algebras (Springer-Verlag), 1971.
3. M. Takesaki: Theory of operator algebras I (Springer-Verlag), 1979.

### **M558: Representations of Linear Lie Groups**

Introduction to topological group, Haar measure on locally compact group, Representation theory of compact groups, Peter Weyl theorem, Linear Lie groups, Exponential map, Lie algebra, Invariant Differential operators, Representation of the group and its Lie algebra. Fourier analysis on  $SU(2)$  and  $SU(3)$ . Representation theory of Heisenberg group . Representation of Euclidean motion group.

*References:*

1. J. E. Humphreys, Introduction to Lie algebras and representation theory. Graduate Texts in Mathematics, 9. Springer-Verlag, 1978.
2. S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari, A first course on representation theory and linear Lie groups. University Press, 2000.
3. Mitsou Sugiura, Unitary Representations and Harmonic Analysis.
4. Sundaram Thangavelu, Harmonic Analysis on the Heisenberg Group, Progress in Mathematics.
5. Sundaram Thangavelu : An Introduction to the Uncertainty Principle: Hardy's Theorem on Lie Groups by, Progress in Mathematics.

### **M557: Representation Theory of Compact Groups**

Locally compact groups, Examples of various matrix groups, Existence of Haar measure (without proof), Computation of Haar measure on  $R, T, SU(2), SO(3)$  and some simple matrix groups, Convolution, the Banach algebra  $L^1(G)$ , General properties of representations of a locally compact group, Complete reducibility, Basic operations on representations, Irreducible representations, Representations of Finite groups, Decomposition of regular representations, Orthogonal relations, Irreducible representations of the symmetry group, Representations of Compact groups, Matrix coefficients, Schur's orthogonality relations, Finite dimensionality of irreducible representations of compact groups, Arzela-Ascoli Theorem, Compact operators, various forms of Peter-Weyl theorem, Character of a representation, Schur's orthogonality relations among characters, Weyl's Character formula, Computing all the irreducible representations of  $SU(2), SO(3)$ .

#### *References:*

1. T. Brocker & T. Dieck: Representations of Compact Lie Groups (Springer), 1985.
2. J. L. Clerc: Les representatios des groupes compacts, Analyse harmonique (J. L. Clerc et al., ed.), C.I.M.P.A., 1982.
3. M. Sugiura: Unitary Representations and Harmonic Analysis (John-Wiley), 1975.
4. E. B. Vinberg: Linear representations of groups (Birkhauser-Verlag), 1989.