

On conservation laws with discontinuous flux

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Abstract: Conservation laws with discontinuous flux appears in the models of two phase flow in porous media, traffic flow with discontinuous road surface, clarifier thickener models of continuous sedimentation, enhanced oil recovery process etc. In this talk we begin with an introduction to both theoretical and numerical aspects of scalar conservation laws with discontinuous flux (CL-DF)[1, 2, 4, 6]. Apart from the basic difficulties for the mathematical analysis, this discussion include the convergence analysis of a second order scheme to the physically relevant (entropy) solution[3]. We continue the discussion with the applications of CL-DF to the system of non strictly hyperbolic partial differential equations, where we propose an efficient numerical method which overcomes the difficulties in the discretization [7]. Together with the stability analysis, this method is applied to a system of equations which models the multicomponent polymer flooding problem of enhanced oil recovery process. In the latter half we discuss a high order numerical method of discontinuous Galerkin scheme applied to a coupled two phase flow-transport problem in the context of discontinuous flux [5]. Apart from this, prior to the summary and future work we discuss about the instability issue which arises in the Buckley-Leverett problem[8].

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