EXTREMAL FUNCTION FOR THE SINGULAR MOSER - TRUDINGER INEQUALITY

Moser- Trudinger inequality is generalised by Adimurthi-Sandeep to the following singular version: Let $\Omega \subset \mathbb{R}^2$ be bounded and $\alpha \in (0, 4\pi], \beta \in [0, 2)$ then

$$\sup_{u \in W_0^{1,2}(\Omega), \int_\Omega |\nabla u|^2 \leq 1} \int_\Omega \frac{e^{\alpha u^2}}{|x|^\beta} dx < \infty \quad \Longleftrightarrow \quad \frac{\alpha}{4\pi} + \frac{\beta}{2} \leq 1.$$

$W_0^{1,2}(\Omega)$ denotes the usual Sobolev space. $\beta = 0$ corresponds to the case of usual Moser-Trudinger inequality. We will discuss the question whether the supremum is attained in the above inequality. If time permits, I will address some issues related to the higher dimension analogue of this inequality.

This is joint work with G. Csato.